

$$\lambda = 2d \sin \theta \qquad \cdots (1)$$

λ: X-RAY WAVELENGTH

d: LATTICE SPACING

 θ : BRAGG'S DIFFRACTION ANGLE

$$\frac{\partial d}{d} = -\cot \theta \ \partial \theta \qquad \dots (2)$$

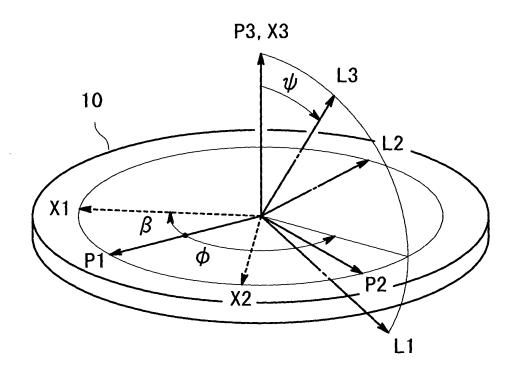
$$\varepsilon = \frac{d - d_0}{d_0} \qquad \cdots (3)$$

 ε : STRAIN

d₀: LATTICE SPACING IN NON-STRAIN STATE

$$\varepsilon = -\cot\theta_0 (\theta - \theta_0) \qquad \dots (4)$$

FIG. 3



P: SPECIMEN COORDINATE SYSTEM

X: CRYSTAL COORDINATE SYSTEM

L: LABORATORY COORDINATE SYSTEM

FIG. 4

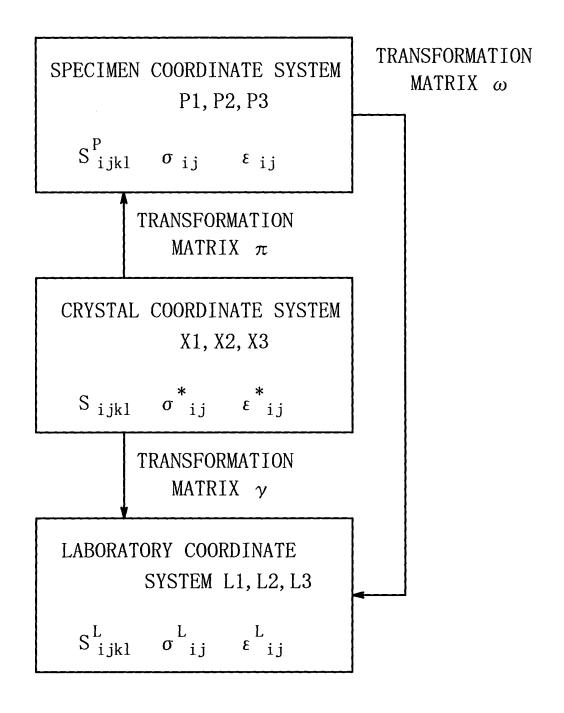


FIG. 5

	S	σ	8
CRYSTAL COORDINATE SYSTEM	S _{ijkl}	σ* ij	* ⁸ ij
SPECIMEN COORDINATE SYSTEM	S ^P ijkl	σ _{ij}	ε _{ij}
LABORATORY COORDINATE SYSTEM	S ^L ijkl	σ ^L ij	L ε ij

S: ELASTIC COMPLIANCE CONSTANT

 σ : STRESS

ε: STRAIN

FIG. 6

ELASTIC COMPLIANCE CONSTANT IN TENSOR NOTATION

S
$$ijkl$$
 (i, j, k, 1 = 1, 2, 3)

RELATIONSHIP

6×6 MATIRIX IN MATRIX NOTATION

$$S_{pq}$$
 (p, q = 1, 2, 3, 4, 5, 6)

i j kl	11	22	33	23	32	13	31	12	21
p q	1	2	3	4	4	5	5	6	6

	p = 1, 2, 3	p = 4, 5, 6	
q = 1, 2, 3	$S_{ijkl} = S_{pq}$	$S_{ijkl} = \frac{1}{2} S_{pq}$	
q = 4, 5, 6	$S_{ijkl} = \frac{1}{2} S_{pq}$	$S_{ijkl} = \frac{1}{4} S_{pq}$	

$$\pi = R3(-\beta) \qquad ...(5)$$

$$\omega = R2(-\psi)R3(-\phi) \qquad ...(6)$$

$$\gamma = \omega \pi \qquad ...(7)$$

$$R1(\delta) = \begin{pmatrix} 1 & 0 & 0 & & & \\ 0 & \cos \delta & -\sin \delta & & \\ 0 & \sin \delta & \cos \delta \end{pmatrix} \qquad ...(8)$$

$$R2(\delta) = \begin{pmatrix} \cos \delta & 0 & \sin \delta & & \\ 0 & 1 & 0 & & \\ -\sin \delta & 0 & \cos \delta \end{pmatrix} \qquad ...(9)$$

$$R3(\delta) = \begin{pmatrix} \cos \delta & -\sin \delta & 0 & & \\ \sin \delta & \cos \delta & 0 & & \\ 0 & 0 & 1 \end{pmatrix} \qquad ...(10)$$

$$\stackrel{L}{\epsilon_{33}} = \gamma_{3i} \gamma_{3j} \stackrel{\epsilon}{\epsilon_{ij}} \qquad ...(11)$$

$$\stackrel{*}{\epsilon_{ij}} = S_{ijkl} \stackrel{*}{\sigma_{kl}} \qquad ...(12)$$

$$\stackrel{*}{\sigma_{kl}} = \pi_{pk} \pi_{ql} \sigma_{pq} \qquad ...(13)$$

$$\stackrel{L}{\epsilon_{33}} = \gamma_{3i} \gamma_{3j} S_{ijkl} \pi_{pk} \pi_{ql} \sigma_{pq} \qquad ...(14)$$

FIG. 8

TETRAGONAL SYSTEM WITH LAUE SYMMETRY 4/mmm

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \cdots (15)$$

TETRAGONAL SYSTEM WITH LAUE SYMMETRY 4/m

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{11} & S_{13} & 0 & 0 & -S_{16} \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ S_{16} - S_{16} & 0 & 0 & 0 & S_{66} \end{bmatrix} \cdots (16)$$

FIG. 9

$$\sigma_{11} = \sigma_{22} = \sigma \qquad \cdots (17)$$

$$\sigma_{12} = \sigma_{13} = \sigma_{23} = \sigma_{33} = 0 \qquad \cdots (18)$$

$$\epsilon_{33}^{L} = (S_{11} + S_{12} - 2S_{13}) \sigma \sin^{2} \psi + 2S_{13} \sigma \cdots (19)$$

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0 \qquad \cdots (20)$$

 $-8S_{13}-S_{66})\,\sigma_{22}+(2S_{11}-2S_{12}-S_{66})\,(\,\sigma_{11}-\sigma_{22}\,)\cos 4\,\beta$ $\epsilon_{33}^{L}(0^{\circ}) = \frac{1}{8} \{ (6S_{11} + 2S_{12} - 8S_{13} + S_{66}) \sigma_{11} + (2S_{11} + 6S_{12}) \sigma_{11} \}$ \cdots (21) - $2(S_{11} - 2S_{12} - S_{66}) \sigma_{12} \sin \theta \beta \sin^2 \psi$ + S_{13} (σ_{11} + σ_{22}) When $\phi = 0^{\circ}$

 $-8S_{13}+S_{66})\,\sigma_{22}-(2S_{11}-2S_{12}-S_{66})\,(\,\sigma_{11}-\sigma_{22}\,)\cos 4\,\beta$ $\epsilon_{33}^{L}(90^{\circ}) = \frac{1}{8} \{ (2S_{11} + 6S_{12} - 8S_{13} - S_{66}) \sigma_{11} + (6S_{11} + 2S_{12}) \sigma_{11} + (6S_{11} + S_{12}) \}$...(22)+ $2(S_{11} - 2S_{12} - S_{66}) \sigma_{12} \sin 4 \beta \sin^2 \psi$ $+ S_{13} (\sigma_{11} + \sigma_{22})$ When $\phi = 90^{\circ}$

When
$$\phi = 45^{\circ}$$

 $\epsilon_{33} (45^{\circ}) = S_{13} (\sigma_{11} + \sigma_{22}) + \frac{1}{8} \{4(S_{11} + S_{12} - 2S_{13}) + (\sigma_{11} + \sigma_{22}) + 2(2S_{11} - 2S_{12} + S_{66}) \sigma_{12} \cos 4\beta$
 $\epsilon_{11} + \sigma_{22} + 2(2S_{11} - 2S_{12} + S_{66}) (\sigma_{11} - \sigma_{22}) \sin 4\beta \} \sin^{2} \psi \dots (2)$

When
$$\phi = 0^{\circ}$$

 $\frac{L}{\epsilon \, 33} \, (0^{\circ}) = \frac{1}{8} \{ (6S_{11} + 2S_{12} - 8S_{13} + S_{66}) \, \sigma_{11} + (2S_{11} + 6S_{12} - 8S_{13} - S_{66}) \, \sigma_{22} + (2S_{11} - 2S_{12} - S_{66}) \, (\sigma_{11} - \sigma_{22}) \cos 4 \, \beta \}$

$$= \sin^2 \psi + S_{13} \, (\sigma_{11} + \sigma_{22}) \qquad \dots (24)$$

When
$$\phi = 90^{\circ}$$

 $\frac{L}{\varepsilon \, 33} (90^{\circ}) = \frac{1}{8} \{ (2S_{11} + 6S_{12} - 8S_{13} - S_{66}) \, \sigma_{11} + (6S_{11} + 2S_{12} - 8S_{13} + S_{66}) \, \sigma_{22} - (2S_{11} - 2S_{12} - S_{66}) \, (\sigma_{11} - \sigma_{22}) \cos 4 \, \beta \}$
 $\sin^2 \psi + S_{13} \, (\sigma_{11} + \sigma_{22})$...(25)

 $\frac{L}{\epsilon_{33}(45^{\circ})} = S_{13}(\sigma_{11} + \sigma_{22}) + \frac{1}{8} \{4(S_{11} + S_{12} - 2S_{13}) + (\sigma_{11} + \sigma_{22}) + 2(2S_{11} - 2S_{12} + S_{66})\sigma_{12}$ + $2(2S_{11} - 2S_{12} - S_{66}) \sigma_{12} \cos 4\beta \sin^2 \psi$ When $\phi = 45^{\circ}$

F1 =
$$\left(\frac{L}{\epsilon_{33}(0^{\circ})} + \frac{L}{\epsilon_{33}(90^{\circ})}\right) / 2$$

= $\frac{1}{2} \left(S_{11} + S_{12} - 2S_{13} \right) \left(\sigma_{11} + \sigma_{22} \right) \sin^2 \psi + S_{13} \left(\sigma_{11} + \sigma_{22} \right)$
...(27)

F2 =
$$\left(\frac{L}{\epsilon_{33}(0^{\circ})} - \frac{L}{\epsilon_{33}(90^{\circ})}\right)/2$$

= $(\sigma_{11} - \sigma_{22})V$

F3 =
$$\frac{L}{\epsilon_{33}} (45^{\circ}) - F1$$

= $2 \sigma_{12} V$

$$V = \frac{1}{8} \{ 2S_{11} - 2S_{12} + S_{66} + (2S_{11} - 2S_{12} - S_{66}) \cos 4 \beta \} \sin^2 \psi$$
 ...(30)

FIG. 15

LAUE SYMMETRY 4/mmm

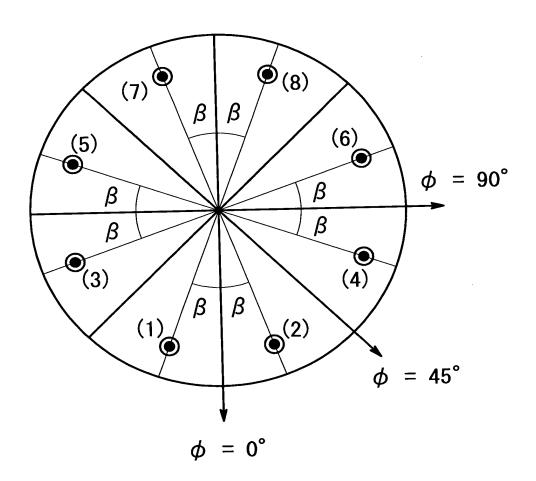


FIG. 16

(4) (5) (6) (7) (8) $\beta - \frac{\pi}{2}$ $\beta + \frac{\pi}{2}$ $-\beta - \frac{\pi}{2}$ $-\beta + \pi$ $\beta - \pi$ $\beta - \frac{3\pi}{4}$ $\beta + \frac{\pi}{4}$ $-\beta - \frac{3\pi}{4}$ $-\beta + \frac{3\pi}{4}$ $\beta - \frac{5\pi}{4}$ $\beta - \frac{3\pi}{4}$	
$\beta + \frac{\pi}{2} - \beta - \frac{\pi}{2} - \beta + \pi \qquad \beta - \tau$ $\beta + \frac{\pi}{4} - \beta - \frac{3\pi}{4} - \beta + \frac{3\pi}{4} \beta - \tau$ $\beta - \beta - \pi - \beta + \frac{\pi}{2} \beta - \tau$	(2) (3)
$\beta + \frac{\pi}{4} - \beta - \frac{3\pi}{4} - \beta + \frac{3\pi}{4} \beta - \frac{3\pi}{4}$ $\beta - \beta - \pi - \beta + \frac{\pi}{2} \beta - \frac{3\pi}{4}$	$-\beta$ $-\beta + \frac{\pi}{2}$
β $-\beta-\pi$ $-\beta+\frac{\pi}{2}$ $\beta-$	$-\beta - \frac{\pi}{4}$ $-\beta + \frac{\pi}{4}$
	$-\beta - \frac{\pi}{2}$

FIG. 17

ψ (°)	β (°)	d ₀ (nm)	θ ₀ (°)
0.00	0.00	0. 2078	21. 76
46.81	0.00	0. 2845	15. 71
36. 98	45.00	0. 1668	27. 50
46.81	0.00	0. 1422	32. 79
67. 22	26. 57	0. 5151	28.60
56. 42	45.00	0. 2299	19. 58
19. 55	0.00	0. 1305	36. 16
56. 42	45. 00	0. 1149	42.08
72.62	0.00	0. 1241	38. 36
	0. 00 46. 81 36. 98 46. 81 67. 22 56. 42 19. 55 56. 42	0.00 0.00 46.81 0.00 36.98 45.00 46.81 0.00 67.22 26.57 56.42 45.00 19.55 0.00 56.42 45.00	0.00 0.00 0.2078 46.81 0.00 0.2845 36.98 45.00 0.1668 46.81 0.00 0.1422 67.22 26.57 0.5151 56.42 45.00 0.2299 19.55 0.00 0.1305 56.42 45.00 0.1149

FIG. 18

